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Analytical one-dimensional model to study the ultrasonic precursor generated by a laser

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We present an analytical one-dimensional model of the laser generation of ultrasound that takes the optical penetration effect into account. This model leads to a simple expression relating the full width at half maximum of the precursor to the product of the optical absorption coefficient by the longitudinal velocity, and establishes the conditions under which this expression is valid. The results of this model are compared to those provided by a more sophisticated semianalytical three-dimensional model and to experimental data obtained under various experimental conditions. The agreement is very good.

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In the thermoelastic regime, the localized temperature elevation due to the absorption of a laser pulse by a sample induces a localized thermal expansion which, in turn, generates ultrasound. The nature of the ultrasonic wave forms depends on the surface distribution and size of the laser beam, on the thermal and optical properties of the material, and on whether the irradiated surface is constrained or not. Many authors studied the phenomenon of the laser generation of ultrasound, beginning with White [1]. Hutchins [2] presented an extensive review of both experimental and theoretical works on the subject. But few authors took the optical penetration effect into account, as most of them concentrated on the laser generation of ultrasound in metallic samples [3–5].

In the case of nonmetals, the penetration of the laser light inside the material may be important, and this phenomenon must be taken into account. Several authors [6–10] developed analytical models of the laser generation of ultrasound including optical penetration effects. However, none of them tried to relate quantitatively the features of the first longitudinal arrival (generally called the precursor) to the optical absorption coefficient. Lyamshev and Chelnokov [10] concentrated on the effects of the optical penetration, the thermal conduction, and the laser pulse duration. They wrote a very general analytical expression for the stress field within the sample but did not carry out any numerical calculations to exploit these mathematical expressions. Tam [11] observed experimentally a temporal broadening of the precursor with increasing optical penetration, but gave only an approximate relation between the full width at half maximum (FWHM) of the precursor, the optical penetration depth and the pulse duration parameter.

Presently, there is a growing interest in the laser generation of ultrasound in nonmetals [12] and especially in the graphite-carbon-epoxy composite [13–15]. In order to understand better the role of buried thermal expansion sources (due to the penetration of the light inside the material) in the ultrasound generation process, we have developed an analytical one-dimensional (1D) model in which the temperature elevation field is assumed to be dictated by the optical absorption only, and gives rise to ultrasonic waves by thermal expansion. This model leads to an expression relating the FWHM of the precursor to the optical absorption coefficient, the longitudinal velocity of the material, and the laser pulse duration parameter. Despite the simplicity of the model, the expression obtained agrees very well with the results of a more sophisticated semianalytical 3D model [16,17] and with experimental data obtained on samples having various optical penetration depths.

The sample of our 1D model is an infinite plate of finite thickness L made of an orthotropic material. The x axis, in the direction of the thickness, is assumed to be one of the three principal axes of the material. At $t=0$, a radiative flux impinges uniformly on the top surface (of equation $x=0$) of the sample. The mechanical displacement field in the x direction $u(x,t)$ is dictated by the following 1D wave equation:

$$\frac{\partial^2 u(x,t)}{\partial x^2}-\frac{1}{v^2}\frac{\partial^2 u(x,t)}{\partial t^2}=\chi\frac{\partial T(x,t)}{\partial x}\ , \tag{1}$$

where v is the longitudinal velocity in the x direction, χ an "apparent" thermal expansion coefficient taking the anisotropy of the material into account [$\chi=(3\lambda+2\mu)\alpha/(\lambda+2\mu)$ in the particular case of an isotropic material, where λ and μ are its Lamé coefficients and α its linear thermal expansion coefficient], and $T(x,t)$ the temperature elevation field imposed by the penetration of the light inside the material. This field is defined in the following way:

$$T(x,t)=\frac{\beta I_0}{\rho C_p}e^{-\beta x}H(t)\ , \tag{2}$$

where β is the optical absorption coefficient, I_0 the energy per surface unit absorbed by the sample, ρ the density, C_p the specific heat, and $H(t)$ the integral over time from 0 to t of the normalized temporal shape $f(t)$ of the laser pulse.

The general solution of Eq. (1) is composed of the general D'Alembert solution of the corresponding homogeneous wave equation, plus a particular solution $u_0(x,t)$:

$$u(x,t)=F(x-vt)+G(x+vt)+u_0(x,t)\ . \tag{3}$$

It is possible to find a particular solution $u_0(x,t)$ having the form $-Ae^{-\beta x}W(t)$ where A is a constant defined by

$$A=\frac{\chi I_0}{\rho C_p}\ . \tag{4}$$

The function $W(t)$ must verify the following differential equation:

$$\frac{1}{v^2}\ddot{W}(t)-\beta^2W(t)=-\beta^2H(t)\ , \tag{5}$$

where the double overdot indicates a double differentiation with respect to time. Moreover, if we impose $W(0)$ and $\dot{W}(0)$ to be zero, the function $W(t)$ is completely determined.

The initial conditions

$$u(x,0)=0 \text{ and } \frac{\partial u}{\partial t}(x,0)=0 \quad \text{for } 0\le x\le L\ , \tag{6}$$

plus the boundary conditions at the two unconstrained surfaces

$$\frac{\partial u}{\partial x}(0,t)-\chi T(0,t)=0 \text{ and } \frac{\partial u}{\partial x}(L,t)-\chi T(L,t)=0 \quad \text{for } t\ge 0\ , \tag{7}$$

allow us to determine the functions $F(x-vt)$ and $G(x+vt)$. The initial conditions, along with the fact that $W(0)$ and $\dot{W}(0)$ are zero, lead to the conclusion that

$$F(x)=K \text{ and } G(x)=-K \quad \text{for } 0\le x\le L\ , \tag{8}$$

where K is a constant. The boundary conditions, along with the fact that $W(t)$ verifies Eq. (5), yield the following conclusions:

$$\begin{aligned}F'(-vt)+G'(vt)&=-A\frac{\ddot{W}(t)}{\beta v^2} \quad \text{for } t\ge 0\ ,\\ F'(L-vt)+G'(L+vt)&=-Ae^{-\beta L}\frac{\ddot{W}(t)}{\beta v^2} \quad \text{for } t\ge 0\ ,\end{aligned} \tag{9}$$

where the prime indicates the derivatives of $F(x-vt)$ and $G(x+vt)$ with respect to the variables $(x-vt)$ and $(x+vt)$, respectively. Equations (9) show that the functions F and G are defined on intervals of lengths L. We are interested only in the solution $u(x,t)$ for $x=L$ and for $0\le vt\le 2L$, which means that we need to determine F and G only on $[-L,L]$ and $[L,3L]$ respectively. Equations (8) and (9) lead to the following expressions for the displacement $u(L,t)$:

$$\begin{aligned}u(L,t)&=-Ae^{-\beta L}\left[W(t)+\frac{\dot{W}(t)}{\beta v}\right] \quad \text{for } 0\le t\le \frac{L}{v}\ ,\\ u(L,t)&=-Ae^{-\beta L}\left[W(t)+\frac{\dot{W}(t)}{\beta v}\right]+2A\frac{\dot{W}(t-L/v)}{\beta v} \quad \text{for } \frac{L}{v}\le t\le\frac{2L}{v}\ .\end{aligned} \tag{10}$$

If we consider the normalized temporal shape of the laser pulse $f(t)$ to be equal to

$$f(t)=\frac{t}{\tau^2}e^{-t/\tau}\ , \tag{11}$$

which is an adequate approximation for most Q-switched lasers (τ multiplied by 2.44639 is the FWHM of the laser pulse), the solution $W(t)$ of Eq. (5) is

$$W(t)=1-\frac{e^{\beta vt}}{2(\beta v\tau+1)^2}-\frac{e^{-\beta vt}}{2(\beta v\tau-1)^2}+\frac{\beta^2v^2\tau e^{-t/\tau}[t(1-\beta^2v^2\tau^2)+\tau(3-\beta^2v^2\tau^2)]}{(\beta^2v^2\tau^2-1)^2} \tag{12}$$

and the displacement $u(L,t)$ becomes

$$u(L,t)=Ae^{-\beta L}\left[\frac{e^{\beta vt}+\beta ve^{-t/\tau}[t(1+\beta v\tau)+\tau(2+\beta v\tau)]}{(1+\beta v\tau)^2}-1\right] \quad \text{for } 0\le t\le\frac{L}{v}\ , \tag{13}$$

$$u(L,t)=Ae^{-\beta L}\left[\frac{\beta ve^{-t/\tau}[t(1+\beta v\tau)+\tau(2+\beta v\tau)]}{(1+\beta v\tau)^2}-1\right]+A\left[\frac{e^{\beta(L-vt)}}{(\beta v\tau-1)^2}+2\beta ve^{(L/v-t)/\tau}\frac{(\beta^2v^2\tau^2-1)(t-L/v)-2\tau}{(\beta^2v^2\tau^2-1)^2}\right] \quad \text{for } \frac{L}{v}\le t\le\frac{2L}{v}\ . \tag{14}$$

We want to study absorbing samples for which $\beta L\ge 5$. We are interested in the precursor, which occurs at times $t\approx L/v$. Thus $\beta vt\approx\beta L\ge 5$. Concerning τ, the value of this parameter is generally much smaller than L/v, which means that $t\gg\tau$. These two remarks allow us to neglect many terms in expressions (13) and (14): defining the new variable $y=\beta(vt-L)$ and the parameter $k=\beta v\tau$, the displacement $u(L,t)$ is a function of y only and takes the simplified form

$$u(y)=\begin{cases} A\dfrac{e^{y}}{(1+k)^2} & \text{for } -\beta L \le y \le 0 \\ A\left[\dfrac{e^{-y}}{(k-1)^2}+2e^{-y/k}\dfrac{(k^2-1)y-2k}{(k^2-1)^2}\right] & \\ & \text{for } 0\le y \le \beta L \end{cases} \quad (15)$$

In the particular case $k=0$ (i.e., $\tau=0$: the temporal shape of the laser pulse is a Dirac function), it is easy to find, using expressions (15), that the maximum of the function $u(y)$ occurs for $y_{max}=0$ (i.e., $t_{max}=L/v$), that its value is A and that its FWHM, $\Delta y_{1/2}=\beta v \Delta t_{1/2}$, is ln(4).

In the general case $k>0$, the analytical inversion of (15) to find the position and value of the maximum and the FWHM is impossible. To observe the behaviors of these parameters, we solved the equations $u'(y_{max})=0$ and $u(y_i)=u(y_{max})/2$ numerically. The curves obtained for y_{max} (the value for which the maximum occurs) and $u(y_{max})$ (the amplitude of this maximum) as functions of k are shown in logarithmic scales in Fig. 1, and the curve obtained for the FWHM $\Delta y_{1/2}=y_2-y_1$ is shown in logarithmic scales in Fig. 2. To these curves, we superpose data obtained with a semianalytical 3D model [16,17]. Another plot of the FWHM curve over the range $0\le k \le 10$ is shown in linear scales in Fig. 3.

It has been possible to plot experimental points on the FWHM curve. That has not been the case for the $u(y_{max})$ curve because of the values of the parameters A of our experiments which were unknown. The confrontation of the y_{max} curve with experimental points has not been possible either due to errors in absolute time measurements introduced by our detection system.

Experiments were performed on two kinds of materials and with two laser sources. In the first series of experiments, a pulsed Nd:YAG laser (monomode, beam radius 1 mm, energy up to 100 mJ, pulse half-width duration 12 ns) was used to generate ultrasound in 3 mm thick BG-18, BG-39, KG-3, and KG-5 *Schott* glasses. The optical absorption values of these glasses were drawn from Ref. [18]. Another experiment was performed with a pulsed CO_2 laser (multimode, rectangular surface profile 4×8 mm^2 in size, energy up to 400 mJ, pulse half-width duration 115 ns) and a 2 mm thick graphite-epoxy composite sample, the optical absorption coefficient of which was measured using the photoacoustic spectroscopy technique [19]. In all these experiments, the normal displacements were measured with an optical heterodyne probe *Ultra-Optec* OP-35.

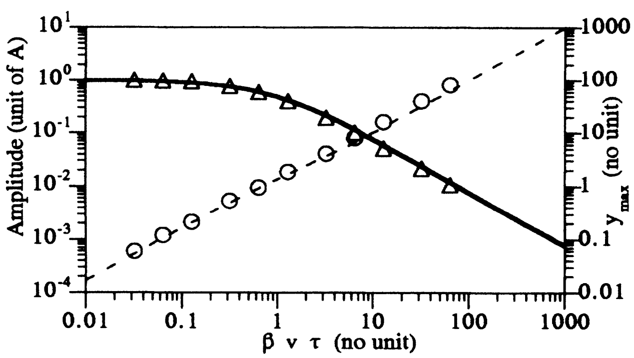

FIG. 1. Amplitude of the maximum of the precursor and position of this maximum as functions of k. The values of the maximum are given in units of A. Amplitude curve: ——, analytical model; Δ, numerical points. y_{max} curve: ----, analytical model; ○, numerical points.

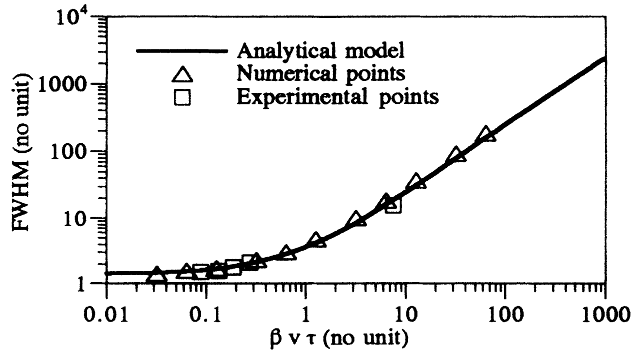

FIG. 2. FWHM of the precursor as a function of k.

Figure 1 shows that $y_{max}\approx k$ over a large range of k values ($0.01\le k \le 1000$), which means that we can approximate very accurately the time for which the precursor reaches its maximum by $L/v+\tau$. This result is easy to interpret: it corresponds to the time for which the power of the laser pulse reaches its maximum plus the delay of acoustic propagation through the sample.

On Figs. 2 and 3, two behaviors of the FWHM as a function of k appear: (i) For $k\le 4$, the linear equation $\Delta y_{1/2}=\ln(4)+2.22k$ fits well the curve given by the analytical model. Over this range of k values, the FWHM of the precursor $\Delta t_{1/2}$ may then be approximated by

$$\Delta t_{1/2}=\frac{\ln(4)}{\beta v}+2.22\tau , \quad (16)$$

which shows that two effects compete to produce the FWHM: the optical penetration (characteristic time $1/\beta v$) and the laser pulse duration (characteristic time τ).

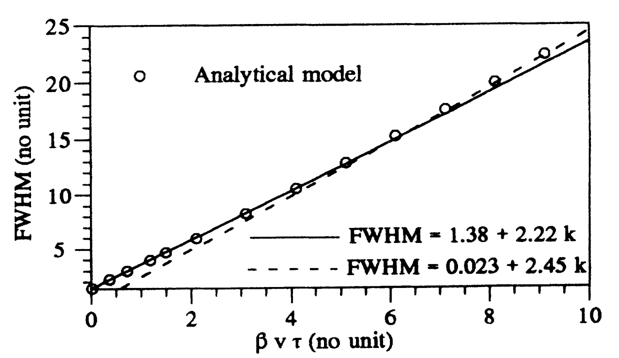

FIG. 3. Enlargement of Fig. 2 in linear scales over the range $0\le k\le 10$.

(ii) For $k \geq 6$, the linear equation $\Delta y_{1/2} = 2.45k$ fits well the curve given by the analytical model: we have then $\Delta t_{1/2} = 2.45\tau =$ the FWHM of the laser pulse in the case of the temporal profile (11) that we have chosen. In that case, the optical penetration has a negligible effect on the FWHM of the precursor, which is essentially related to the temporal profile of the laser pulse.

It is interesting to note that the behavior of the amplitude as a function of k (Fig. 1) is inverse to the one of the FWHM (Fig. 2): approximately constant and equal to A for $k \leq 0.06$ and proportional to $1/k$ for $k \geq 10$. In fact, the product of the amplitude by the FWHM is nearly constant over the range of k values that we have considered. This result can be associated with a conservation of energy: for the same laser pulse energy and for the same sample, the product of $\Delta t_{1/2}$ by the amplitude of the precursor is nearly independent of the laser pulse duration.

We have presented experimental data obtained with samples having optical absorption coefficients ranging from 3000 to 50000 m^{-1} and with two different excitation sources. The experimental results were always in very good agreement with our simple analytical 1D model, even when the irradiation experimentally performed could not be considered as uniform. The respect of the condition of uniform irradiation is related to the directivity pattern of the sample. In the case of our *Schott* glasses, their large optical penetration depths make the condition of uniform irradiation not crucial. However, the respect of this condition may be more critical for other materials.

In conclusion, we have shown that the FWHM of the precursor is governed by two effects: optical penetration and laser pulse duration. For short laser pulses, the FWHM is proportional to the time of acoustic propagation through the thickness of the thermal source $1/\beta$, while for long laser pulses, it is equal to the FWHM of the laser pulse. In the case of short laser pulses, it is therefore possible to relate the absolute value of the optical absorption coefficient to the FWHM of the precursor. This will allow us to measure this coefficient quantitatively by the laser-ultrasonics technique, respecting certain conditions of laser pulse duration and of irradiation. Combined with a photoacoustic experiment allowing the quantitative measurement of the product of the optical absorption ceofficient by the square root of the thermal diffusivity [19], we will be able to perform optical and thermal characterizations of materials.

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